OptimalSlope V 0.6
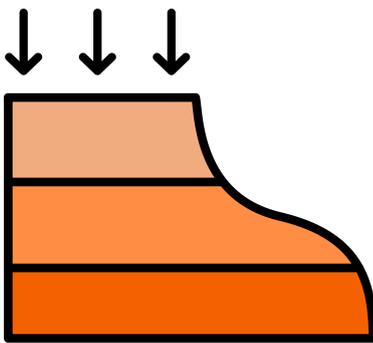## Cloud computing version

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Introduction

The steepness of the slopes of an open pit mine has a substantial influence on the financial return of the mine. In current practice pitwall profiles are planar in cross-section. Sometimes rock layers exhibiting different strengths require the inclination of the profile to vary, but the inclination across each layer is constant.

Instead OptimalSlope provides depth varying optimal pitwall profiles. OptimalSlope seeks the solution of a mathematical optimisation problem where the overall steepness of the pitwall, from crest to toe, is maximized for an assigned stratigraphy, rock properties and Factor of Safety (FoS). Bench geometries (bench height, face inclination, minimum width) are imposed in the optimization as constraints which bind the maximum local inclination of the sought optimal profile together with any other constraints such as geological discontinuities that may influence slope failure. The optimal profiles are always steeper than their planar counterparts (i.e. the profile exhibiting the same FoS) up to 8 degrees depending on rock type and severity of constraints.

The adoption of optimal slope profiles in an open pit mine always leads to reduction of rockwaste volume that translate into higher Net Present Value and reductions of carbon footprint and energy consumption.

1 Historical background on optimal profiles

Anecdotal evidence of the fact that a slope profile non-linear in cross-section, i.e. a profile whose inclination varies with depth, is better than a linear one, i.e. a planar profile, was reported as far back as 1890. In fact, Newman (1890) observed that cuttings of concave shape excavated in homogeneous clay layers tended to be more stable than planar ones with the same OSA which in turn are more stable than cuttings of convex shape. Many decades later, Hoek & Bray in chapter 12 of the second edition of 'Rock slope engineering' analyse the stability of some concave circular slopes in cross-section. Assuming the slopes to be excavated in a homogeneous rock layer and using the Mohr-Coulomb (M-C) failure criterion to characterize its strength, i.e. cohesion, c, and internal friction angle, ϕ, they found a higher stability number (which is a dimensionless index capturing the mechanical stability of a slope) for circular profiles than for their planar counterparts, i.e. the planar slopes with the same OSA which share the same toe and crest points. They also mention that circular shapes had been considered by Rana & Bullock (1969) for the design of the pitwalls of an iron pit mine in Canada. However, Hoek & Bray strongly caution against the assumption of uniform slope since it completely disregards the fact that the strength of geomaterials tends to vary with depth, being usually lower in the upper part of the excavated slope due to well-known geological processes (e.g. soil deposition, weathering, etc.). The first systematic theoretical study on the mechanical properties of concave slope profiles for geomaterials exhibiting some cohesion, so applicable to all rocks and clayey soils, appeared in Utili & Nova (2007). In that study, the superior stability of logarithmic spiral (logspiral) profiles, with the logspiral being featured by a radius of curvature increasing with the depth of excavation, was systematically proven over their planar counterparts. By employing the limit analysis upper

bound method, Utili & Nova first systematically determined the optimal logspiral shape, i.e. the shape associated to the highest stability number for several prescribed OSAs, second compared the optimal logspiral slopes to their planar counterparts. They show that logspiral profiles exhibit higher FoS than their planar counterparts for any value of c, and ϕ considered with the highest gain for inclinations midway ϕ and the vertical line, i.e. for $OSA \sim \pi/4 + \phi/2$. Since then, other researchers (Jeldes et al. 2015; Vahedifard et al. 2016; and Vo & Russell 2017) have independently investigated the stability of $c - \phi$ concave profiles, employing different methods, namely the slip line method (Jeldes et al. 2015), limit equilibrium methods (Vahedifard et al. 2016) and the finite element method for the assessment of slope stability. They all reached the same conclusion concerning the superior stability of non-linear concave profiles. However, a key limitation of these studies is the assumption of a specific shape, either a circle (Vahedifard et al. 2016) or a log-spiral (Utili & Nova 2007) or a curve stemming from the slip-line field theory and the associated characteristic equations (Jeldes et al. 2015), so that the shape claimed to be optimal is found as the shape associated to the highest stability number among curves belonging to a very restricted family. It is obvious that these profiles are sub-optimal and their shape could be very different from the truly optimal one. In case of $c - \phi$ slopes, the optimal shape calculated by OptimalSlope turns out to be partly concave and partly convex so significantly different from the purely concave shapes considered in previous literature. Another perhaps even more important limitation resides in the assumption of uniform slope present in all the aforementioned methods that prevents the application of these findings to real open pit mines which are typically featured by complex stratigraphies involving multiple layers of different mechanical strengths and various geological discontinuities.

The search for the optimal shape of a slope is a difficult problem of topological optimization since slopes develop very significant irrecoverable (plastic) deformations which cause significant stress redistribution to occur before reaching failure, but the theory of topological optimization developed so far in engineering deals predominantly with elastic media (Bendsoe & Sigmund, 2004). Neglecting the onset of plastic deformations in a slope, i.e. assuming a purely elastic behavior, is not a viable option since this would severely underestimate the slope resistance to failure hence the slope Factor of Safety. And the application of plasticity theory to topological optimization is only in its infancy (Fin et al., 2019). Therefore current algorithms in the literature for topological optimization are not viable to search for the optimal profile of slopes. The code OptimalSlope exploits the fact that slope failures occur either as a rotational mechanism (a planar failure being a particular type of rotational failure with an infinite radius of curvature) or mechanisms whose kinematics is dictated by the presence of discontinuities, e.g. the interface between two rock layers, a fault, joints, beddings etc.. For a homogeneous slope in a $c - \phi$ geomaterial, the limit analysis upper bound method allows to find the critical rotational mechanism by determining the minimum of an analytical objective function without requiring any discretization of the slope into finite elements (Chen, 1975). The function is obtained imposing the energy balance between the external work done by the mass of the candidate failure mechanism and the energy dissipated along its failure surfaces. The equation has been extended to find the critical mechanism for piecewise linear slope profiles in a uniform layer and then to the case of layered slopes. Also, the formulation has been extended to slopes in rocks obeying the generalized Hoek-Brown (G-H-B) failure criterion (Hoek, 1994). The minimum of the function stemming from the energy balance equation and therefore the critical mechanism is found by OptimalSlope. Since the time the G-H-B criterion was introduced, a strong consensus has gathered in the rock mechanics community that it is a better criterion than M-C to describe the strength of rock masses. But for highly weathered rocks and residual soils M-C is better (Martin & Stacey, 2018), so there are several cases of mines where the M-C criterion is employed to fit the rock strength data of some layers whilst the G-H-B criterion for some other layers. OptimalSlope can deal with these situations too, i.e. some rock layers being characterized by the M-C criterion whilst others by the G-H-B criterion, since the G-H-B parameters of any layer are converted by OptimalSlope into equivalent $c - \phi$ parameters, employing the method of Renani & Martin (2019).

In the excavation of an open pit, several rock layers of different strength are usually encountered. OptimalSlope can find the optimal profile for any specified lithological sequence (any number of layers can be specified as input with each layer strength characterized by either M-C or G-H-B parameters) without unduly restricting the search to any predefined family of shapes. The optimal slope profile is found as the solution of a mathematical optimisation problem where the OSA of the slope, i.e. the inclination from slope crest to toe, is maximized for an assigned stratigraphy, rock strength properties and a prescribed FoS. Geometric requirements stemming from bench sizes (bench height, face inclination and minimum berm width) are imposed as constraints binding the maximum local inclination of the sought optimal profile together with any other geometric constraint, e.g. constraints to prevent the occurrence of local failure mechanisms due to geological discontinuities such as faults and joints.

1.1 References

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3 Launching OptimalSlope

OptimalSlope is a compiled MATLAB application verified to work on Microsoft Windows, macOS and Linux.

Operating Systems	
Windows (all versions from Windows 7 on are supported)	
Linux (all version and flavours)	
macOS (all versions)	
Internet connection	
An internet connection is necessary to connect to the computational Server	
Processors	
Any Intel or AMD x86-64 processor	
Disk	
Minimum	Recommended
3.5 GB of HDD space for MATLAB runtime	An SSD is recommended
RAM	
4 gb	
Graphics	
No specific graphics card is required.	

Table 1: System Requirements

3.1 System Requirement

OptimalSlope is a compiled MATLAB application which runs on an remote cloud computing server. This server utilises the MATLAB Runtime to execute MATLAB code free of charge. The recommended system requirement to access OptimalSlope are listed in Table 1.

3.2 Getting Started

3.2.1 Install OptimalSlope

Installation of OptimalSlope it is not necessary since the software is available from the cloud computing server.

3.2.2 Launch the application

The procedure to launch the OptimalSlope is similar between Windows and Unix systems. Simply open a command line window (or terminal window on Unix systems) and type:

```
Command Line
ssh -t USER@CLOUD_SERVER_IP /usr/OptimalSlope/application/run_OptimalSlope.sh
/usr/local/MATLAB/MATLAB_Runtime/v98
```

The server will ask to insert the PASSWORD, and immediately start OptimalSlope.

3.2.3 Usage of OptimalSlope

Once the software is launched correctly, the user will see the following welcome screen:

```
Command Line
   ____       _   _              _ _____ _
  / __ \     | | (_)           | |/ ____| |
 | |  | |_ __ | |_ _ _ __ ___   __ _| | (___ | | ___  _ __   ___
 | |  | | '_ \| __| | '_ ` _ \ / _` | |\___ \| |/ _ \| '_ \ / _ \
 | |__| | |_) | |_| | | | | | | (_| | |____) | | (_) | |_) |  __/
  \____/| .__/ \__|_|_| |_| |_|\__,_|_|_____/|_|\___/| .__/ \___|
        | |                                          | |
        |_|                                          |_|
 Software for the determination of OPTIMAL profiles for SLOPEs and pitwalls
 Copyright: all rights reserved
 Version 0.6 - March 2021
```

3.2.4 Operating the software

The operation of OptimalSlope is completely based on the text interface GUI. Right after the welcome screen, the user is prompt with a request to input the email address where OptimalSlope will send the simulation results. This is the only way that the software has to provide outputs to the user, therefore OptimalSlope asks confirmation of the e-mail address before proceeding with the input of the simulation.

```
Command Line
 Provide an email address to send the simulation results: test@test.com
 Is test@test.com correct (T,F)?
```

◆

Notice: OptimalSlope will send an email to defined address with the following details:

- **address**: optimalslope@gmail.com

- **subject**: OptimalSlope simulation results

- **main body**: This message was automatically generated by OptimalSlope software. This email address is not monitored. Regards. OptimalSlope Team

- **attachments**: Results.zip. This compressed folder will contain the results of the simulation, which are:

  - picture in .svg format representing the optimal profile (Figure **??**a), the critical failure surface determined by OptimalSlope and the computational limits for the optimal slope shape (UB and LB). The name of the picture contains the scenario computed, the Overall Slope Angle (measured toe-crest) of the optimal Slope profile and the predicted Factor of Safety.

  - simulation output in .txt format with the OSA, adimensional coordinates of the optimal slope profile, predicted FoS, target FoS and computational time to obtain the final optimal slope profile. The name of the picture contains the scenario computed, the Overall Slope Angle (measured toe-crest) of the optimal Slope profile and the predicted Factor of Safety.

  - command window .txt file with all the input data and the iterations performed to get the final optimal profile.

If the option T is selected, OptimalSlope asks the geotechnical input simulation parameters. To date, only 2 failure criteria (Mohr-Coulomb and Generalised-Hoek-Brown) are considered to define the geomaterial strength, but more are coming soon.

For each geomaterial it is also asked the definition of the maximum value of bench face angle and berm width, which must be computed beforehand with bench stability analyses and the rockfall analyses.

```
Input simulation parameters...

    - Overall slope height [m]: VALUE
    - Number of segments defining the optimal slope profile
        (number of benches): VALUE
     -> The bench height is: X m
    - Target FoS: VALUE
    - Number of different Generalised-Hoek-Brown geomaterials: VALUE
        Unconfined Compressive Strenght i [mPa]: VALUE
        Geological Strenght Index i: VALUE
        Material constant for the intact rock (mi) i: VALUE
        Damage factor i: 1
        Soil unit weight 1 [kN/m3]: VALUE
        Maximum confinement stress to calculate the equivalent Mohr-Coulomb
            parameters [mPa], if the Renani & Martin relationship has to be
            used input NaN i: VALUE
        bench face angle i [deg]: VALUE
        berm width i [m]: VALUE
    - Number of different Mohr-Coulomb geomaterials: VALUE
        phi i+1 [deg]: VALUE
        c i+1 [kPa]: VALUE
        unit weight i+1 [kN/m3]: VALUE
        bench face angle i+1 [deg]: VALUE
        berm width i+1 [m]: VALUE
```

Later, OptimalSlope asks to define how the geomaterials defined above are distributed from the topographic surface and it will show the options available. If the user defined only one geomaterial, OptimalSlope will assume that the slope under design is homogeneous and won't show this menu.

```
    Define how the materials are distributed starting from topographic surface
    (materials available: i  i+1  i+2):
        Material for the layer going from dZ_1 m to dZ_2 m: VALUE
        Material for the layer going from dZ_2 m to dZ_3 m: VALUE
        Material for the layer going from dZ_3 m to dZ_4 m: VALUE
        Material for the layer going from dZ_4 m to dZ_5 m: VALUE
```

Successively OptimalSlope asks the `Overall Slope Angle planar case`, which is the starting point for an iterative procedure used to obtain FoS(`OSA optimal profile`) = `target FoS`. This value should be taken from the corresponding planar slope profile case where FoS(`OSA planar case`) = `target FoS`.

```
    - Overall Slope Angle planar case [deg]: VALUE
```

Later, OptimalSlope asks the `road width`. If the user wants to model an optimal slope profile without roads, `NaN` must be inserted as `road width`.

```
    - Road width (set NaN for no road) [m]: VALUE
```

If a value of `road width` is inserted, OptimalSlope asks the vertical position of the road measured from the crest of the slope. If the user wants to insert more than one road the values of `road position` must be enclosed by squared brackets. On the contrary, if the user set `NaN`, as `road width`, OptimalSlope won't show this option.

```
      - Road position (vertical distance from the crest - values beween [ ]
        for more roads): VALUE
```

Afterwards, OptimalSlope ask if the user wants to apply a distributed surcharge on the crest of the slope. In case of positive answer (y) a submenu appears asking to insert the parameters defining the surcharge, that are:

- The `unit weight of the surcharge` (γ_s), which is used to model the condition presented in Figure 1, and it represents the unit weight of the soil modelled with a trapezoidal surcharge.

- The `inclination of the triangular component of the distributed load` (α), which is represented in figure 1.

- The `height of the surcharge` (H_s), which is used to model the condition presented in Figure 1, and it represents the height of the soil modelled with a trapezoidal surcharge.

- The `distance between the crest and the surcharge` (D), which represent the eccentricity of the trapezoidal surcharge measured from the crest of the slope (Figure 1).
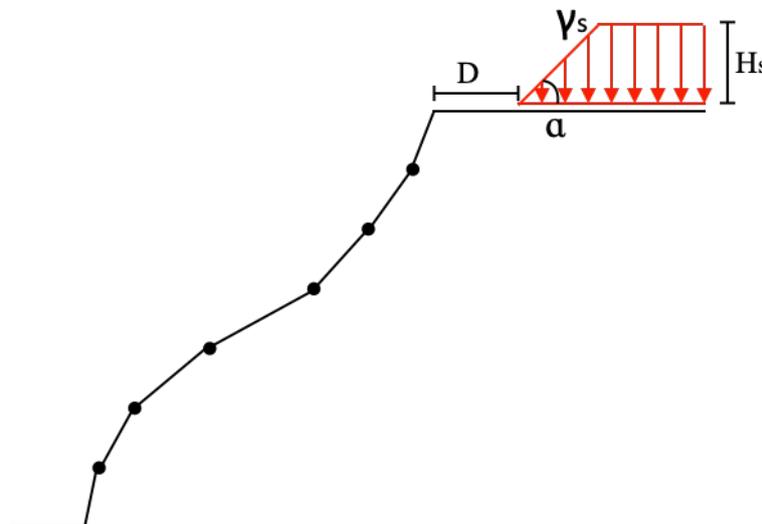

Figure 1: Geometry surchcarge

```
      Do you want to apply a distributed surcharge? [y/n] y
      - Unit weight of the distributed surcharge [kN/m3]: VALUE
      - Inclination of the triangular component of the distributed surcharge
          [deg]: VALUE
      - Height of the distributed surcharge [m]: VALUE
      - Distance between the crest and the distributed surcharge [m]: VALUE
```

Finally, OptimalSlope asks if the user want to set some non default settings. For now, the only non default settings available are:

- The choice to save ptimal profiles produced in the search for the optimal profile with the defined `target FoS`.

- The `maximum number of iterations` used to reach the `target FoS`.

- The number of `physical cores` allocated for the current simulation. This number represents how many physical cores that will be used in the parallelisation stage of the software. OptimalSlope suggest, as number `physical cores`, the number of physical CPU cores that MATLAB Runtime can use on the machine where OptimalSlope is installed. Once the number of workers is defined, OptimalSlope starts a parallel pool used for the following computations.

- The `starting point`, which represents the starting profile for the optimisation procedure which computes the optimal slope profile. If 'planar' is selected, the software selects the most planar profile which is respecting the constraints on the `face_angle` and `road_angle`. Instead if 'non-linear' is selected, the software uses a non-linear slope profile, which is computed by linear interpolation of known optimal slope profiles. This profile is also respecting the constraints defined before.

More additional settings will be added soon.

Command Line

```
     Do you want to set additional options? [y/n] y
         - Save all the optimal profiles computed? [y/n] n
         - Maximum number of iterations used to reach the target FoS
             (suggested value = 3): VALUE
         - Physical cores used for the computation of the optimal profile
             (suggested number X): VALUE
         - Starting point (planar or non-linear): VALUE
```

Finally OptimalSlope will ask confirmation for the data set as input and if the user select the option `T`, will begin the search of the optimal profile with the input parameters defined above.

Command Line

```
  Confirm data inserted (T,F):
```